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On branes ending on branes in supergravity

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Abstract: We study the eleven-dimensional supergravity and the classical solutions corresponding to M2-branes ending on M5-branes. We obtain the generic BPS configuration representing two sets of parallel M5-branes with $(1+1)$ commonly longitudinal directions and M2-branes stretching between them. We also discuss how the brane creation is described in supergravity.

Keywords: p-branes, Black Holes in String Theory.

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Contents

1. Introduction

Branes or the spatially extended solitons play the central role in the study of string theory or M-theory. They are known to form a variety of bound states, and the corresponding classical solutions should exist in the effective supergravity theories. However, a difficulty arises in constructing classical solutions corresponding to the bound states of intersecting branes. In the most simple descriptions of intersecting branes in terms of harmonic functions [1, 2], any branes are delocalized along the directions longitudinal to the other branes. (There are a few exceptions such as [3] and [4].) BPS saturated configurations have also been known for partially localized intersection of branes, or two types of branes intersecting each other with only one of which delocalized along the other. Recently a fully localized BPS configurations have been found for M5-branes intersecting on three-branes [5] and for M2-brane junctions [6, 7]. The results have also been generalized to various dual configurations [6]. In this paper we try to obtain another BPS configurations for localized branes intersecting one another.

In the above we mean by the term "localized" that the distribution of branes is not uniform. Generically the supergravity fields for BPS saturated bound states of branes are expressed in terms of some arbitrary functions. From these functions one can read off the distribution of the source branes, and they have to satisfy certain equations of motion in order to describe delta-functional sources. For fully localized intersection of branes the equations of motion are highly complicated and difficult to solve. We will encounter the same difficulty in this paper.

In this paper we shall work in the eleven-dimensional supergravity [8]. It contains a three-form potential besides the graviton and the gravitino, and M2-branes and M5-branes are the electric and magnetic sources of the potential. The presence of a single BPS M2-brane or M5-brane preserves a half of supersymmetry. An M2-brane and an M5-brane are known to form a 1/4-supersymmetric bound state when they intersect on a string. It is also believed that an M2-brane can end on an M5-brane. In this case the M2-brane is "open" and has a $(1+1)$-dimensional boundary on the M5-brane. Interestingly, such open M2-branes can be created between two M5-branes with $(1+1)$ commonly longitudinal directions when the two M5-branes pass through each other. Such creation of branes are familiar in the D-brane worldvolume gauge field theory, and a supergravity analysis has been made for a specific example [9].

In the following we shall find the generic BPS configuration corresponding to the bound states of M2-branes(012), M5-branes(013456) and M5-branes(01789$\natural$). The numbers specify the directions the branes are lying along, and we denoted the eleventh direction by $\natural$. The analysis of the BPS condition $\delta_{\rm SUSY}\,\psi_m = 0$ is performed in section 2 for spherical symmetric configurations, and the result is fully generalized in section 3. The resultant expression depends on three arbitrary functions of $x_{2,3,\dots,9,\natural}$. These functions have to satisfy the equations of motion given in section 4 in order to describe a bound state of localized(delta-functional) branes. They are not solved, but we show that if the M5-branes are localized, the boundaries of M2-branes stretching between them are automatically localized. We also propose in section 5 an idea for understanding the brane creation in supergravity.

2. Analysis of the BPS condition

Here we analyze the BPS condition under the assumption of spherical symmetry. The solution is summarized in (2.23) and (2.24), and the following arguments help the reader to understand to what extent the result is general.

We parameterize 01-directions by t, σ, 2-direction by x_2, 3456-directions by $x_1, \theta_{\rm I}, \phi_{\rm I}, \psi_{\rm I}$ and 789$\natural$-directions by $x_3, \theta_{\rm II}, \phi_{\rm II}, \psi_{\rm II}$. We assume the following form for the metric:

$$\begin{aligned} ds^2 &= g_0^2\left(-dt^2 + d\sigma^2 + \varpi^1\varpi^1 + \varpi^2\varpi^2 + \varpi^3\varpi^3\right) + g_{\rm I}^2\, d\Omega_{\rm I}^2 + g_{\rm II}^2\, d\Omega_{\rm II}^2\,, \\ d\Omega_i^2 &= d\theta_i^2 + \cos^2\theta_i\, d\phi_i^2 + \sin^2\theta_i\, d\psi_i^2\,, \quad i = {\rm I} \text{ or } {\rm II}\,, \end{aligned} \tag{2.1}$$

where $\varpi^{1,2,3}$ are linear combinations of $dx^{1,2,3}$ with coordinate-dependent coefficients. We define the vielbein as follows:

$$\begin{array}{llll} e^{\underline{t}} = g_0\, dt\,; & e^{\underline{1}} = g_0\, \varpi^1\,; & e^{\underline{\theta_{\rm I}}} = g_{\rm I}\, d\theta_{\rm I}\,; & e^{\underline{\theta_{\rm II}}} = g_{\rm II}\, d\theta_{\rm II}\,, \\ e^{\underline{\sigma}} = g_0\, d\sigma\,; & e^{\underline{2}} = g_0\, \varpi^2\,; & e^{\underline{\phi_{\rm I}}} = g_{\rm I} \cos\theta_{\rm I}\, d\phi_{\rm I}\,; & e^{\underline{\phi_{\rm II}}} = g_{\rm II} \cos\theta_{\rm II}\, d\phi_{\rm II}\,, \\ e^{\underline{3}} = g_0\, \varpi^3\,; & e^{\underline{\psi_{\rm I}}} = g_{\rm I} \sin\theta_{\rm I}\, d\psi_{\rm I}\,; & e^{\underline{\psi_{\rm II}}} = g_{\rm II} \sin\theta_{\rm II}\, d\psi_{\rm II}\,. & \end{array}$$

Hereafter we denote local Lorentz indices with underbar. Under the spherical symmetry, the most generic form for the four-form field strength is given by:

$$
\begin{aligned}
F_{(4)} &= F^{[M2]}_{(4)} + F^{[M5]}_{(4)} + F^{[M5']}_{(4)} \\
&= g_0^{-1} \left(\frac{1}{2}\, e^{\underline{t\sigma ij}}\, E_k\, \epsilon^{ijk} + e^{\underline{\theta_{\mathrm{II}}\phi_{\mathrm{II}}\psi_{\mathrm{II}} i}}\, B_i + e^{\underline{\theta_{\mathrm{I}}\phi_{\mathrm{I}}\psi_{\mathrm{I}} i}}\, H_i \right) \\
&= dt\, d\sigma \wedge \frac{1}{2}\, g_0^3\, \epsilon^{ijk}\, E_i\, \varpi^{jk} + d^3\Omega_{\mathrm{II}} \wedge g_{\mathrm{II}}^3\, B_i\, \varpi^i + d^3\Omega_{\mathrm{I}} \wedge g_{\mathrm{I}}^3\, H_i\, \varpi^i\,.
\end{aligned}
\tag{2.2}
$$

Here the overall factor g_0^{-1} in the second line is simply for later convenience.

To obtain supersymmetric solutions we have to focus on the supersymmetry transformation law of gravitino

$$
e_{\underline{m}}{}^{m}\, \delta_\epsilon\, \psi_m = e_{\underline{m}}{}^{m}\, \mathcal{D}_m\, \epsilon - \frac{1}{288}\left(3\, \Gamma^{\underline{pqrs}}\, \Gamma_{\underline{m}} - \Gamma_{\underline{m}}\, \Gamma^{\underline{pqrs}}\right) \epsilon F_{\underline{pqrs}}\,, \tag{2.3}
$$

and find field configurations that admit $\delta_\epsilon \psi_m = 0$ for some non-zero ϵ. Here and throughout this section we use "mostly hermitean" Gamma matrices satisfying

$$
\begin{aligned}
\left\{\Gamma^{\underline{a}}, \Gamma^{\underline{b}}\right\} &= 2\, \eta^{\underline{ab}} = 2\, \mathrm{diag}(- + + \cdots +)\,, \\
\Gamma^{\underline{a_1 \cdots a_{11}}} = \epsilon^{a_1 \cdots a_{11}}\,, &\qquad \epsilon^{t\,\sigma 123 \theta_{\mathrm{I}}\phi_{\mathrm{I}}\psi_{\mathrm{I}}\theta_{\mathrm{II}}\phi_{\mathrm{II}}\psi_{\mathrm{II}}} = 1\,.
\end{aligned}
$$

Let us evaluate (2.3) term by term. To begin with, the covariant derivative of a spinor is defined by

$$
e_{\underline{m}}{}^{m} \mathcal{D}_m \epsilon = e_{\underline{m}}{}^{m} \left(\partial_m + \frac{1}{4}\, \Omega_{m\underline{pq}}\, \Gamma^{\underline{pq}} \right) \epsilon \equiv \left(\nabla_{\underline{m}} + \frac{1}{4}\, \Omega_{\underline{mpq}}\, \Gamma^{\underline{pq}} \right) \epsilon\,; \quad \nabla_{\underline{m}} \equiv e_{\underline{m}}{}^{m}\, \partial_m\,, \tag{2.4}
$$

where $\Omega_{\underline{pq}} = dx^m\, \Omega_{m\underline{pq}}$ is the spin connection. Under the assumption of spherical symmetry we can obtain some components of the spin connection from the torsion-free condition alone:

$$
\mathcal{D} e^{\underline{p}} = de^{\underline{p}} + \Omega^{\underline{p}}{}_{\underline{q}}\, e^{\underline{q}} = 0\,. \tag{2.5}
$$

The only non-zero components of $\Omega_{\underline{mnp}}$ except for those with $(m, n, p = 1, 2, 3)$ are

$$
\begin{aligned}
-\Omega_{\underline{tti}} = \Omega_{\underline{\sigma\sigma i}} &= \nabla_{\underline{i}} \ln g_0\,, \\
\Omega_{\underline{\theta_{\mathrm{I}}\theta_{\mathrm{I}} i}} = \Omega_{\underline{\phi_{\mathrm{I}}\phi_{\mathrm{I}} i}} = \Omega_{\underline{\psi_{\mathrm{I}}\psi_{\mathrm{I}} i}} &= \nabla_{\underline{i}} \ln g_{\mathrm{I}}\,, \\
\Omega_{\underline{\theta_{\mathrm{II}}\theta_{\mathrm{II}} i}} = \Omega_{\underline{\phi_{\mathrm{II}}\phi_{\mathrm{II}} i}} = \Omega_{\underline{\psi_{\mathrm{II}}\psi_{\mathrm{II}} i}} &= \nabla_{\underline{i}} \ln g_{\mathrm{II}}\,, \\
\Omega_{\underline{\phi_{\mathrm{I}}\phi_{\mathrm{I}}\theta_{\mathrm{I}}}} = -\frac{1}{g_{\mathrm{I}}} \frac{\sin\theta_{\mathrm{I}}}{\cos\theta_{\mathrm{I}}}\,; &\qquad \Omega_{\underline{\psi_{\mathrm{I}}\psi_{\mathrm{I}}\theta_{\mathrm{I}}}} = \frac{1}{g_{\mathrm{I}}} \frac{\cos\theta_{\mathrm{I}}}{\sin\theta_{\mathrm{I}}}\,, \\
\Omega_{\underline{\phi_{\mathrm{II}}\phi_{\mathrm{II}}\theta_{\mathrm{II}}}} = -\frac{1}{g_{\mathrm{II}}} \frac{\sin\theta_{\mathrm{II}}}{\cos\theta_{\mathrm{II}}}\,; &\qquad \Omega_{\underline{\psi_{\mathrm{II}}\psi_{\mathrm{II}}\theta_{\mathrm{II}}}} = \frac{1}{g_{\mathrm{II}}} \frac{\cos\theta_{\mathrm{II}}}{\sin\theta_{\mathrm{II}}}\,.
\end{aligned}
$$

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Using these expressions we can rewrite the BPS condition $\delta_\epsilon \psi_m = 0$ in the following way:

$$\begin{aligned}
2\,\Gamma^{\underline{\theta_i}}\nabla_{\underline{\theta_i}}\,\epsilon &= 2\,\Gamma^{\underline{\phi_i}}\nabla_{\underline{\phi_i}}\,\epsilon + \Gamma^{\underline{\theta_i}}\epsilon\,\nabla_{\underline{\theta_i}}\ln\cos\theta_i \\
&= 2\,\Gamma^{\underline{\psi_i}}\nabla_{\underline{\psi_i}}\,\epsilon + \Gamma^{\underline{\theta_i}}\epsilon\,\nabla_{\underline{\theta_i}}\ln\sin\theta_i\,, \quad i = \mathrm{I} \text{ or } \mathrm{II}\,,
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
g_0\left(4\nabla_{\underline{i}} + \Omega_{\underline{ijk}}\,\gamma^{jk} - 2\nabla_{\underline{j}}\ln g_0\gamma^i\,\gamma^j\right)\epsilon &= 2\left(E_i\gamma^{123} + B_i\gamma^0 - H_i\gamma^5\right)\epsilon\,, \\
3\,g_0\not{\nabla}\ln g_0\epsilon &= \left(-2\not{E}\gamma^{123} - \not{B}\,\gamma^0 + \not{H}\gamma^5\right)\epsilon\,, \\
6\,g_0\Gamma^{\underline{t\sigma\theta_{\mathrm{I}}}}\nabla_{\underline{\theta_{\mathrm{I}}}}\epsilon + 3\,g_0\not{\nabla}\ln g_{\mathrm{I}}\epsilon &= \left(\not{E}\gamma^{123} - \not{B}\gamma^0 - 2\not{H}\gamma^5\right)\epsilon\,, \\
6\,g_0\Gamma^{\underline{t\sigma\theta_{\mathrm{II}}}}\nabla_{\underline{\theta_{\mathrm{II}}}}\epsilon + 3\,g_0\not{\nabla}\ln g_{\mathrm{II}}\epsilon &= \left(\not{E}\gamma^{123} + 2\not{B}\gamma^0 + \not{H}\gamma^5\right)\epsilon\,.
\end{aligned} \tag{2.7}$$

Here we have introduced a set of new gamma-matrices

$$\left(\gamma^0,\gamma^1,\gamma^2,\gamma^3\right) = \left(\Gamma^{\underline{\theta_{\mathrm{II}}\phi_{\mathrm{II}}\psi_{\mathrm{II}}}},\Gamma^{\underline{t\,\sigma 1}},\Gamma^{\underline{t\,\sigma 2}},\Gamma^{\underline{t\,\sigma 3}}\right)\,, \qquad \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = -\Gamma^{\underline{\theta_{\mathrm{I}}\phi_{\mathrm{I}}\psi_{\mathrm{I}}}} \tag{2.8}$$

and used some short-hand notations

$$\not{\nabla} \equiv \gamma^1\nabla_{\underline{1}} + \gamma^2\,\nabla_{\underline{2}} + \gamma^3\,\nabla_{\underline{3}}\,, \qquad \not{E} \equiv \gamma^1 E_1 + \gamma^2 E_2 + \gamma^3 E_3\,, \text{ etc.} \tag{2.9}$$

γ^0 and γ^5 are anti-hermitean while $\gamma^{1,2,3}$ are hermitean. We also assumed in the above that ϵ does not depend on t and σ.

We would like to obtain the bosonic field configuration that satisfies the BPS condition $\delta_\epsilon\psi_m = 0$ with the following ϵ:

$$\epsilon = f(x_i)\,U_{\mathrm{I}}(\theta_{\mathrm{I}},\phi_{\mathrm{I}},\psi_{\mathrm{I}})\,U_{\mathrm{II}}(\theta_{\mathrm{II}},\phi_{\mathrm{II}},\psi_{\mathrm{II}})\epsilon_0\,, \tag{2.10}$$

where $f(x_i)$ is a scale factor, $U_{\mathrm{I,II}}$ are two mutually commuting local Lorentz transformations and ϵ_0 is a constant spinor. We also assume that the residual supersymmetry is characterized by

$$\Gamma^{\underline{t\,\sigma 2}}\epsilon = \Gamma^{\underline{t\,\sigma 3\theta_{\mathrm{II}}\phi_{\mathrm{II}}\psi_{\mathrm{II}}}}\epsilon = \Gamma^{\underline{t\,\sigma 1\theta_{\mathrm{I}}\phi_{\mathrm{I}}\psi_{\mathrm{I}}}}\epsilon = \epsilon \qquad \text{or} \qquad \gamma^2\epsilon = \gamma^{30}\epsilon = \gamma^{51}\epsilon = \epsilon\,. \tag{2.11}$$

We can rather easily find the solution of the angular equation (2.6) satisfying also (2.11). It is given by the following U_{I} and U_{II}:

$$\begin{aligned}
U_{\mathrm{I}} &= \exp\left(-\frac{\theta_{\mathrm{I}}\Gamma^{\underline{\theta_{\mathrm{I}}1}}}{2}\right)\exp\left(-\frac{\phi_{\mathrm{I}}\Gamma^{\underline{\phi_{\mathrm{I}}1}}}{2}\right)\exp\left(-\frac{\psi_{\mathrm{I}}\Gamma^{\underline{\psi_{\mathrm{I}}\theta_{\mathrm{I}}}}}{2}\right) \\
U_{\mathrm{II}} &= \exp\left(-\frac{\theta_{\mathrm{II}}\Gamma^{\underline{\theta_{\mathrm{II}}3}}}{2}\right)\exp\left(-\frac{\phi_{\mathrm{II}}\Gamma^{\underline{\phi_{\mathrm{II}}3}}}{2}\right)\exp\left(-\frac{\psi_{\mathrm{II}}\Gamma^{\underline{\psi_{\mathrm{II}}\theta_{\mathrm{II}}}}}{2}\right)\,.
\end{aligned} \tag{2.12}$$

These local Lorentz transformations are understood as relating the polar frame (where $\Gamma^{\underline{t,\sigma,1,\ldots,\psi_{\mathrm{II}}}}$ are coordinate independent) to the orthonormal frame (where $\Gamma^{\underline{0,1,\ldots,\natural}}$

are coordinate independent). The remaining equations (2.7) determine the dependence on the coordinates $x_{1,2,3}$. Using (2.10) and (2.11) we can rewrite them as follows:

$$2\nabla_{\underline{i}}\,\epsilon = \nabla_{\underline{i}} \ln g_0\, \epsilon\,, \tag{2.13}$$

$$g_0 \left(\frac{1}{2}\,\Omega_{\underline{ipq}}\,\gamma^{pq} - \nabla_{\underline{p}} \ln g_0\,\gamma^{ip}\right) = E_i\,\gamma^{31} - B_i\,\gamma^{23} + H_i\,\gamma^{12}\,, \tag{2.14}$$

$$E_2 = B_1 = -H_3\,, \tag{2.15}$$

$$\begin{aligned} 3\,g_0 \not\nabla \ln g_0 &= \gamma^1\,(-2E_3 + H_2) + \gamma^2\,(-B_3 - H_1) + \gamma^3\,(B_2 + 2E_1)\,,\\ -3\,g_0\,g_{\mathrm{I}}^{-1}\gamma^1 + 3\,g_0 \not\nabla \ln g_{\mathrm{I}} &= \gamma^1\,(E_3 - 2H_2) + \gamma^2\,(-B_3 + 2H_1) + \gamma^3\,(B_2 - E_1)\,,\\ -3\,g_0\,g_{\mathrm{II}}^{-1}\,\gamma^3 + 3\,g_0 \not\nabla \ln g_{\mathrm{II}} &= \gamma^1\,(E_3 + H_2) + \gamma^2\,(2B_3 - H_1) + \gamma^3\,(-2B_2 - E_1)\,. \end{aligned} \tag{2.16}$$

The first equation (2.13) determines the scale factor of ϵ as follows:

$$\epsilon = g_0^{1/2}\,U_{\mathrm{I}}\,U_{\mathrm{II}}\,\epsilon_0\,. \tag{2.17}$$

The next equations (2.14) and (2.15) relate the components of the spin connection and the gauge field strength:

$$\begin{aligned} &\Omega_{\underline{112}} = g_0^{-1}H_1 + \nabla_{\underline{2}} \ln g_0\,; && \Omega_{\underline{212}} = g_0^{-1}H_2 - \nabla_{\underline{1}} \ln g_0\,; && \Omega_{\underline{312}} = g_0^{-1}H_3\,,\\ &\Omega_{\underline{123}} = -g_0^{-1}B_1\,; && \Omega_{\underline{223}} = -g_0^{-1}B_2 + \nabla_{\underline{3}} \ln g_0\,; && \Omega_{\underline{323}} = -g_0^{-1}B_3 - \nabla_{\underline{2}} \ln g_0\,,\\ &\Omega_{\underline{131}} = g_0^{-1}E_1 - \nabla_{\underline{3}} \ln g_0\,; && \Omega_{\underline{231}} = g_0^{-1}E_2\,; && \Omega_{\underline{331}} = g_0^{-1}E_3 + \nabla_{\underline{1}} \ln g_0\,. \end{aligned}$$

Since the torsion-free condition (2.5) relates the spin connection to the vielbein, the above relations allow us to express the components of the gauge field strength in terms of the vielbein. The most elegant way is:

$$\begin{aligned} d\varpi^1 &= H_1\,\varpi^{21} + E_1\,\varpi^{13}\,,\\ d\varpi^2 &= H_2\,\varpi^{21} + 2\,E_2\,\varpi^{13} + B_2\,\varpi^{23}\,,\\ d\varpi^3 &= B_3\,\varpi^{23} + E_3\,\varpi^{13}\,. \end{aligned} \tag{2.18}$$

The most convenient choice of coordinates that is compatible with the above would be

$$\begin{aligned} \varpi^1 &= h_1 dx^1\,,\\ \varpi^2 &= h_2\left(dx^2 + A_1 dx^1 + A_3 dx^3\right),\\ \varpi^3 &= h_3\,dx^3\,. \end{aligned} \tag{2.19}$$

The next equations (2.16) enable us to express $g_{0,\mathrm{I},\mathrm{II}}$ in terms of the components of ϖ^i. A careful analysis of them with the help of (2.18) and (2.19) yields that

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$g_{\rm I}/h_1 g_0$ depends only on x_1, and similarly $g_{\rm II}/h_3 g_0$ depends only on x_3. Using the diffeomorphism degrees of freedom we can therefore set

$$g_{\rm I} = x_1 h_1\, g_0\,, \qquad g_{\rm II} = x_3\, h_3\, g_0\,.$$

Using the above relation we then find $g_0\, g_{\rm I}\, g_{\rm II} = x_1\, x_3$. Hence

$$g_0 = (h_1 h_3)^{-1/3}\,, \qquad g_{\rm I} = x_1 h_1^{2/3} h_3^{-1/3}\,, \qquad g_{\rm II} = x_3\, h_1^{-1/3} h_3^{2/3}\,. \tag{2.20}$$

The equations (2.16) also yield the following equations:

$$\partial_1 \left(\frac{h_2 h_3}{h_1} \right) = \partial_2 \left(\frac{A_1 h_2 h_3}{h_1} \right), \qquad \partial_3 \left(\frac{h_2 h_1}{h_3} \right) = \partial_2 \left(\frac{A_3 h_2 h_1}{h_3} \right). \tag{2.21}$$

Hence we put

$$h_2^2 = \partial_2 X \partial_2 Y\,, \qquad h_1^2 = \frac{\partial_2 Y}{\partial_2 Z}\,, \qquad h_3^2 = \frac{\partial_2 X}{\partial_2 Z}\,, \qquad A_1 = \frac{\partial_1 X}{\partial_2 X}\,, \qquad A_3 = \frac{\partial_3 Y}{\partial_2 Y}\,. \tag{2.22}$$

Thus the most generic solution of the BPS condition with spherical symmetry is summarized as follows:

$$\begin{aligned} ds^2 = (\partial_2 X \partial_2 Y \partial_2 Z)^{-1/3} \Bigg[& \partial_2 Z \left(-dt^2 + d\sigma^2\right) + \partial_2 Y \left(dx_1^2 + x_1^2 d\Omega_{\rm I}^2\right) + \\ & + \partial_2 X \left(dx_3^2 + x_3^2 d\Omega_{\rm II}^2\right) + \\ & + \partial_2 X \partial_2 Y \partial_2 Z \left(dx_2 + \frac{\partial_1 X}{\partial_2 X} dx_1 + \frac{\partial_3 Y}{\partial_2 Y} dx_3 \right)^2 \Bigg]\,, \end{aligned} \tag{2.23}$$

$$\begin{aligned} F_{(4)} = {} & dt\, d\sigma \wedge \frac{1}{2}\, d\left[dx_1 D_1 Z + dx_3 D_3 Z\right] + \\ & + d^3\Omega_{\rm II} \wedge \frac{1}{2} \left[-d\left(x_3^3 D_3 X\right) + x_3^3 dx_3 \left(\frac{\partial_2}{\partial_2 Y} \frac{\partial_2 X}{\partial_2 Z} + x_3^{-3} D_3\, x_3^3 D_3 X \right) \right] + \\ & + d^3\Omega_{\rm I} \wedge \frac{1}{2} \left[-d\left(x_1^3 D_1 Y\right) + x_1^3 dx_1 \left(\frac{\partial_2}{\partial_2 X} \frac{\partial_2 Y}{\partial_2 Z} + x_1^{-3} D_1 x_1^3 D_1 Y \right) \right]. \end{aligned} \tag{2.24}$$

Here D_1 and D_3 are the "coordinate covariant derivatives" defined as follows:

$$D_1 \equiv \partial_1 - \frac{\partial_1 X}{\partial_2 X}\, \partial_2 \equiv \partial_1|_{X \rm fixed}\,, \qquad D_3 \equiv \partial_3 - \frac{\partial_3 Y}{\partial_2 Y}\, \partial_2 \equiv \partial_3|_{Y \rm fixed}\,.$$

These are obviously invariant under the change of the coordinate $x_2 \rightarrow x_2' = f(x_1, x_2, x_3)$. This is a residual diffeomorphism symmetry, and owing to this symmetry we may parameterize the 2-direction by any of X, Y, Z.

3. Generalization

From the previous result (2.23), (2.24) we can guess the expression for more general solutions without spherical symmetry. It is expressed by three arbitrary functions X, Y, Z of $x_{2,3,\ldots,\natural}$ as follows:

$$ds^2 = (\partial_2 X \partial_2 Y \partial_2 Z)^{-1/3} \Bigg[\partial_2 Z(-dt^2 + d\sigma^2) + \partial_2 Y dx_i^2 + \partial_2 X dx_p^2 + \\ + \partial_2 X \partial_2 Y \partial_2 Z \left(dx_2 + \frac{\partial_i X}{\partial_2 X} dx_i + \frac{\partial_p Y}{\partial_2 Y} dx_p \right)^2 \Bigg] , \qquad (3.1)$$

$$2F_{(4)} = dt\, d\sigma \wedge d\left[dx_i D_i Z + dx_p D_p Z\right] + \\ + \left[\frac{1}{6} \epsilon^{pqrs} d(D_p X) dx_q dx_r dx_s - dx_7 dx_8 dx_9 dx_{10} \left(\frac{\partial_2}{\partial_2 Y} \frac{\partial_2 X}{\partial_2 Z} + D_p D_p X \right) \right] + \\ + \left[\frac{1}{6} \epsilon^{ijkl} d(D_i Y) dx_j dx_k dx_l - dx_3 dx_4 dx_5 dx_6 \left(\frac{\partial_2}{\partial_2 X} \frac{\partial_2 Y}{\partial_2 Z} + D_i D_i Y \right) \right] , \qquad (3.2)$$

$$i, j, k, l = (3, 4, 5, 6) , \qquad p, q, r, s = (7, 8, 9, 10) .$$

The coordinate covariant derivatives are defined as follows:

$$D_i \equiv \partial_i - \frac{\partial_i X}{\partial_2 X} \partial_2 \equiv \partial_i|_{X\,\mathrm{fixed}} , \qquad D_p \equiv \partial_p - \frac{\partial_p Y}{\partial_2 Y} \partial_2 \equiv \partial_p|_{Y\,\mathrm{fixed}} .$$

We can safely say that the above expression is the most generic BPS configuration, because it is the unique generalization of the most generic spherical symmetric configuration obtained in the previous section. We would like to note here again that one of X, Y, Z is a residual diffeomorphism degree of freedom.

4. Equation of motion

The equation of motion in the absence of the source is given by

$$dF_{(4)} = dF_{(7)} - F_{(4)} \wedge F_{(4)} = 0 , \qquad (4.1)$$

$$\frac{1}{4} \left[R_{mn} - \frac{1}{2} g_{mn} R \right] = \frac{1}{12} \left[F_{mpqr} F_n{}^{pqr} - \frac{1}{8} g_{mn} F_{pqrs} F^{pqrs} \right] . \qquad (4.2)$$

A careful analysis of these equations shows that, under the assumption of the BPS condition some of the above equations turn out equivalent. The result is that the solution of (4.1) automatically satisfies (4.2). Therefore we concentrate on (4.1) in the following.

In the presence of the source the equation of motion is modified as

$$dF_{(4)} = j_5 , \qquad dF_{(7)} - F_{(4)} \wedge F_{(4)} = j_8 . \qquad (4.3)$$

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BPS condition now relates the components of the stress tensor to the components of j_5 and j_8. Our generic BPS configuration (3.1), (3.2) has the following currents:

$$2\,j_5 = -d^4x_{789\natural}\wedge df^{[M5]} - d^4x_{3456}\wedge df^{[M5']}\,, \tag{4.4}$$

$$\begin{aligned}2\,j_8 &= -d^8x_{3456789\natural}\, f^{[M2]} - d^4x_{789\natural}\, Dx_2\,\partial_2 Y\wedge \frac{1}{6}\,\epsilon^{ijkl}D_i\, f^{[M5]}\, d^3x_{jkl} - \\ &\quad - d^4x_{3456}\, Dx_2\,\partial_2 X\wedge\frac{1}{6}\,\epsilon^{pqrs}D_p\, f^{[M5']}\, d^3x_{qrs} - \\ &\quad - dt\, d\sigma\, Dx_2\, d^4x_{3456}\wedge \partial_2 Z df^{[M5']} - dt\, d\sigma Dx_2\, d^4x_{789\natural}\wedge\partial_2 Z df^{[M5]}\,, \end{aligned}\tag{4.5}$$

$$Dx_2 \equiv dx_2 + dx_i\,\frac{\partial_i X}{\partial_2 X} + dx_p\,\frac{\partial_p Y}{\partial_2 Y}\,, \qquad d^n x_{j_1\cdots j_n}\equiv dx_{j_1}\cdots dx_{j_n}\,,$$

where the three functions $f^{[M2,M5,M5']}$ are defined as follows:

$$\begin{aligned} f^{[M5]} &= \frac{\partial_2}{\partial_2 Y}\,\frac{\partial_2 X}{\partial_2 Z} + D_pD_pX\,, \\ f^{[M5']} &= \frac{\partial_2}{\partial_2 X}\,\frac{\partial_2 Y}{\partial_2 Z} + D_iD_iY\,, \\ f^{[M2]} &= D_iD_i\,\frac{\partial_2 X}{\partial_2 Z} + D_pD_p\,\frac{\partial_2 Y}{\partial_2 Z} - 2D_iD_pX\,D_pD_iY + \frac{\partial_2}{\partial_2 Y}\,\frac{\partial_2 X}{\partial_2 Z}\cdot\frac{\partial_2}{\partial_2 X}\,\frac{\partial_2 Y}{\partial_2 Z}\,. \end{aligned}\tag{4.6}$$

These encode the position of the sources. The source-free equations of motion are hence given by $f^{[M2]}=f^{[M5]}=f^{[M5']}=0$.

If both M5-branes and M5'-branes are present, they possibly bend each other. However, bending of branes is a notion that depends on the choice of coordinates. We may say that there is no bending effects if we can find in a natural way a coordinate frame in which both M5 and M5'-branes are flat. But the following consideration leads us to conclude that this is not the case.

Looking at the expressions for currents carefully, one finds that the fourth and the fifth terms in j_8 of (4.5) correspond to M2-brane charges with euclidean worldvolume. Hence it is reasonable to require them to vanish even in the presence of the source. We therefore impose the following condition:

$$D_i\, f^{[M5]}\equiv D_p\, f^{[M5']}\equiv 0\,. \tag{4.7}$$

This is equivalent to requiring that $f^{[M5]}$ is a function of (x_p, X) and $f^{[M5']}$ is a function of (x_i, Y). Under the above condition the currents take the following simple form:

$$\begin{aligned} 2\,j_5 &= -d^4x_{789\natural}Dx_2\,\partial_2 f^{[M5]} - d^4x_{3456}\,Dx_2\,\partial_2 f^{[M5']}\,, \\ 2\,j_8 &= -d^8x_{3456789\natural}\, f^{[M2]}\,. \end{aligned}$$

The classical solution for some isolated M5 and M5'-branes is thus obtained by solving

$$
\begin{aligned}
f^{[M5]} &= \frac{\partial_2}{\partial_2 Y}\frac{\partial_2 X}{\partial_2 Z} + D_p D_p X = \sum_j Q_j \delta^4\left(x_p - a_p^{(j)}\right)\theta\left(X - a_2^{(j)}\right), \\
f^{[M5']} &= \frac{\partial_2}{\partial_2 X}\frac{\partial_2 Y}{\partial_2 Z} + D_i D_i Y = \sum_j Q'_j \delta^4\left(x_i - b_i^{(j)}\right)\theta\left(Y - b_2^{(j)}\right).
\end{aligned} \tag{4.8}
$$

The solution corresponds to the system of M5-branes of charge Q_j at $(X, x_p) = (a_2^{(j)}, a_p^{(j)})$ and M5'-branes of charge Q'_j at $(Y, x_i) = (b_2^{(j)}, b_i^{(j)})$. We find that M5-branes are flat in $x_2 = X$ frame while M5'-branes are flat in $x_2 = Y$ frame. Hence we conclude that the M5-branes and M5'-branes in general bend each other.

Choosing one of X, Y, Z as the x_2-coordinate we can regard (4.8) as two equations for two unknown functions. They are non-linear and highly complicated equations, (X, Y, Z) appearing as coordinates as well as functions. Moreover the solution of (4.8) must not be unique because there is a freedom to put an arbitrary number of M2-branes. At present the generic solution for them is not known. It is known, however, that under the assumption

$$
\partial_2 X = H_5\left(x_p\right), \qquad \partial_2 Y = H_{5'}\left(x_i\right), \qquad \left(\partial_2 Z\right)^{-1} = H_2\left(x_i, x_p\right).
$$

the equations of motion are reduced to the following linear differential equations:

$$
\partial_p\,\partial_p\,H_5 = \partial_i\,\partial_i\,H_{5'} = \left(H_5\,\partial_i\,\partial_i + H_{5'}\partial_p\,\partial_p\right)H_2 = 0\,.
$$

This type of equations has been analyzed in [2], [10]–[14] in different contexts. The above equations describe the system of M2-branes together with some M5 and M5'-branes smeared along the x_2-direction. Since all the fields are x_2-independent the solutions cannot represent M2-branes ending on M5-branes.

The third equation $f^{[M2]}\!=\!0$ remains to be analyzed. In analyzing this, recall that one of X, Y, Z is the gauge degree of freedom. Therefore if $f^{[M2]}\!=\!f^{[M5]}\!=\!f^{[M5']}\!=\!0$ were three independent equations, the system would be over-determined. This is not the case. The point is that the x_2-derivative of $f^{[M2]}$ is zero where $f^{[M5]}\!=\!f^{[M5']}\!=\!0$. Indeed, using (4.7) we find

$$
\partial_2\,f^{[M2]} = \partial_2\,f^{[M5]}\left(f^{[M5']} - D_i D_i Y\right) + \partial_2\,f^{[M5']}\left(f^{[M5]} - D_p D_p X\right). \tag{4.9}
$$

Since $\partial_2 f^{[M2]}$ represents the boundaries of M2-branes, the above equality means that M2-branes can have boundaries only on M5-branes.

5. Brane creation

We would like to give an idea for how the brane creation can be seen in supergravity. Let us consider the system of an M5-brane and an M5'-brane. Then the functions

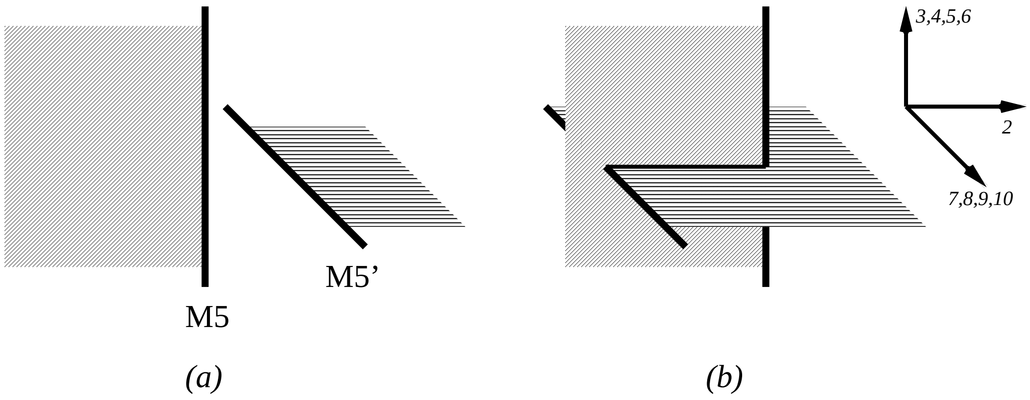

Figure 1: Creation of an M2-brane by an M5 and an M5'-brane passing through each other.

$f^{[M5]}$ and $f^{[M5']}$ have support on semi-infinite six-planes that are bounded by M5- and M5'-branes, respectively. Assume that one of the two six-planes is on the left of the M5-brane, and the other is on the right of the M5'-brane, as depicted in figure 1. Note that one can change whether a six-plane appears on the left or on the right of an M5-brane by the shift $X \to X + f(x_p)$ or $Y \to Y + f(x_i)$. Then, according to the relative position of two M5-branes the two six-planes may or may not have an intersection. Since the component $f^{[M2]}$ of the M2-brane current j_8 satisfies

$$\begin{aligned} \partial_2 f^{[M2]} &= \partial_2 \left(f^{[M5]} f^{[M5']}\right) - \partial_2 f^{[M5]} D_i D_i Y - \partial_2 f^{[M5']} D_p D_p X \\ \text{or} \quad f^{[M2]} &= f^{[M5]} f^{[M5']} + \cdots , \end{aligned} \tag{5.1}$$

there is an M2-brane precisely on the intersection of two six-planes, and its charge is proportional to the product of the charges of the two M5-branes. This explains the brane creation in supergravity, namely, when an M5-brane pass through an M5'-brane, an M2-brane is created between them.

It is expected that all the other types of M2-branes, namely those with semi-infinite worldvolume or those stretching between M5-M5 or M5'-M5' are described by the second and third terms in (5.1).

We give here a simple example. Let us solve the equation of motion under the assumption that X, Y, Z depend only on x_2. The solution representing the system of an M5-brane at $Z = a$ and an M5'-brane at $Z = b$ is obtained by solving

$$\frac{\partial_Z^2 X}{\partial_Z Y} = \theta(Z - a)\,, \qquad \frac{\partial_Z^2 Y}{\partial_Z X} = -\theta(b - Z)\,. \tag{5.2}$$

Assuming $a < b$, the solution is given in terms of a function $f(Z)$ satisfying $f''(Z) = -f(Z)$ as follows:

$$\begin{array}{lll} (Z \le a) & (a \le Z \le b) & (b \le Z) \\ \begin{cases} \partial_Z X = f(a) \\ \partial_Z Y = f''(a)(Z-a) + f'(a) \end{cases} & \begin{cases} \partial_Z X = f(Z) \\ \partial_Z Y = f'(Z) \end{cases} & \begin{cases} \partial_Z X = f'(b)(Z-b) + f(b) \\ \partial_Z Y = f'(b) \end{cases} \end{array} .$$

Then $f^{[M2]}$ takes the following form as expected:

$$f^{[M2]} = \frac{\partial_Z^2 X}{\partial_Z Y} \frac{\partial_Z^2 Y}{\partial_Z X} = -\theta(Z-a)\,\theta(b-Z)\,. \tag{5.3}$$

This represents the M2-brane stretching between the two M5-branes, completely delocalized in the $x_{3,4,\ldots,9,\natural}$-directions. If the right-hand sides of the equations (5.2) are shifted by constants, the solutions will contain some M2-branes with semi-infinite worldvolume. It is straightforward to find such solutions.

6. Conclusion

In this article we have found the most generic BPS configuration for M5-branes (013456), M5'-branes(01789$\natural$) and M2-branes(012). We have also given and studied the equation of motion for localized sources. The equations are highly non linear, and it seems very difficult to obtain the generic solution. In fact, it is not clear whether or not the solution for localized M5- and M5'-branes indeed exists. But the analysis of the equations of motion themselves has lead to some interesting results.

By focusing on a specific term in the M2-brane current we have given an explanation for the brane creation in supergravity. Strictly speaking, however, this is no more than a conjecture because we have no justification for picking up a specific term in the current. Constructing a solution for the equations (4.8) will help us in great deal in understanding the mechanism of brane creation in supergravity and checking if the above conjecture indeed holds.

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